

Advanced Algorithms

April 28, 2026

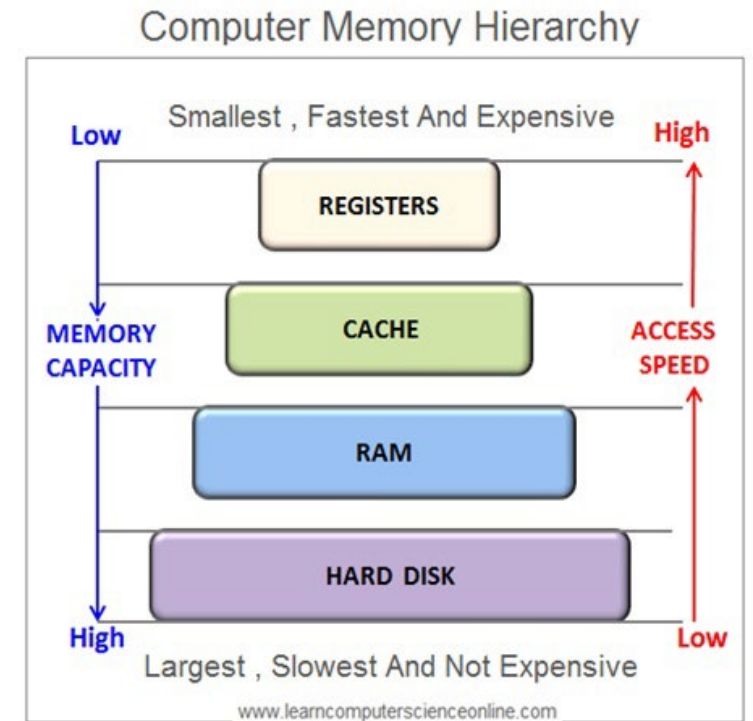
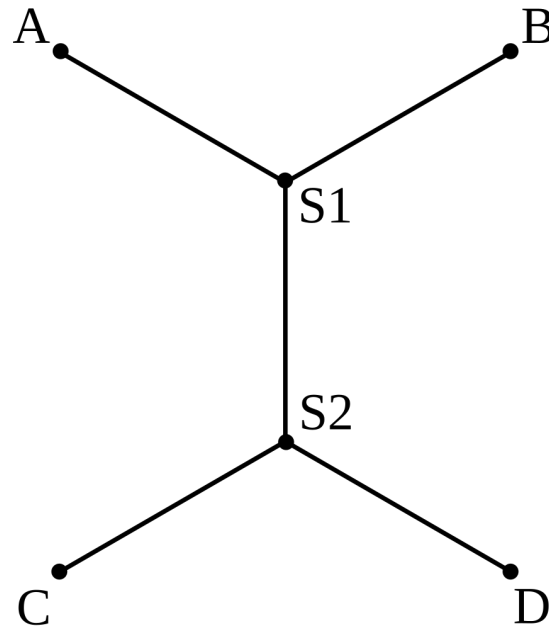
Welcome to the “Final” Class

Today:

- Course evals
- Last week debrief
- Top 10 list!
- Bit about my research / Final Projects AMA
- Will have time after class today to discuss (until 4:00)

Debrief from last week

- Caching comparisons
- Online Steiner Tree



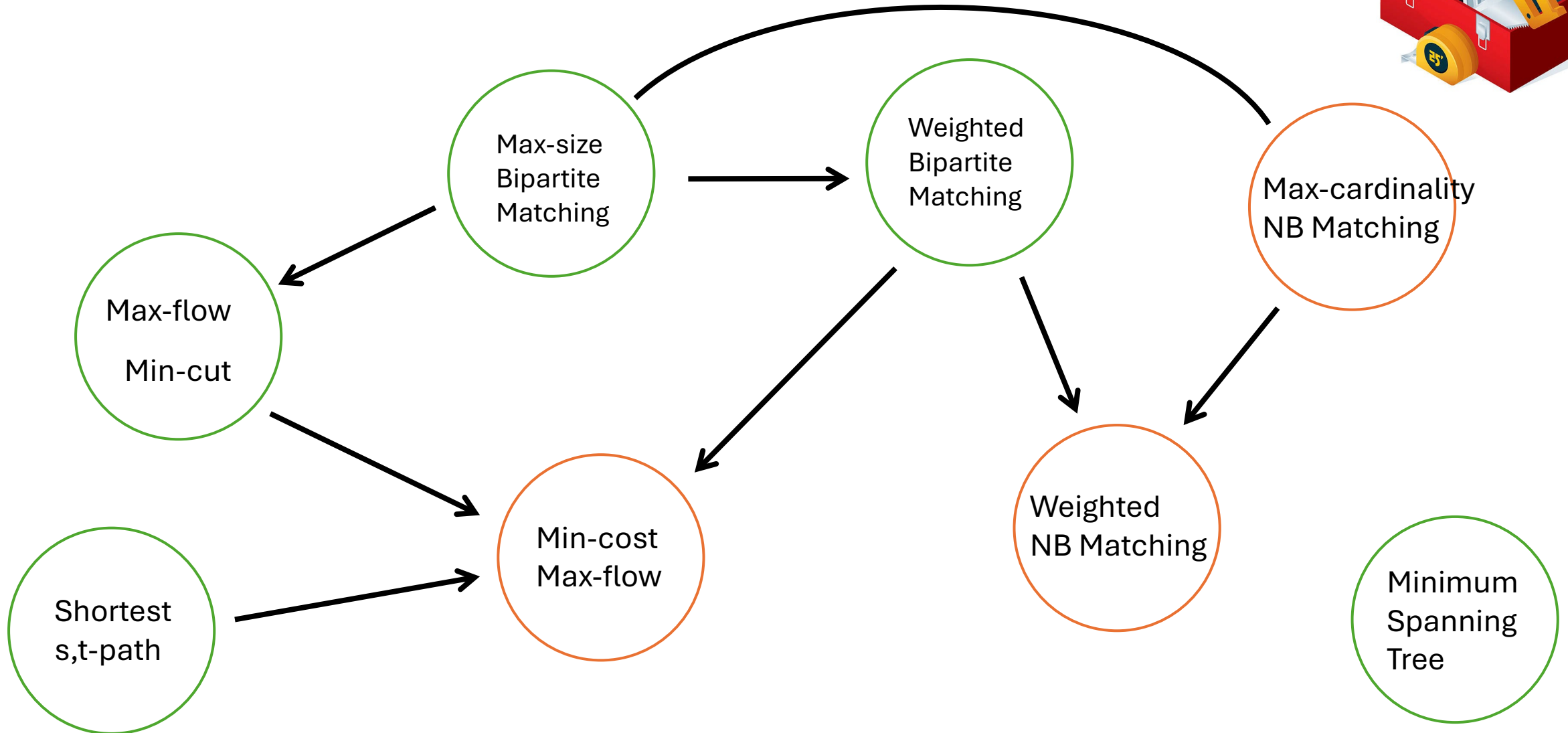
Advanced Algorithms

- Greatest Hits
- Make your own top 3!

THE MATCH[®]
NATIONAL RESIDENT MATCHING PROGRAM[®]

Stable Matchings

Fundamental Problems in P



Linear and Integer Programming

x_A = number of Aqua-Spas to produce

x_H = number of Hydro-Luxes to produce

$$\text{Maximize: } 350x_A + 300x_H$$

Subject to:

$$x_A + x_H \leq 200 \quad (\text{pumps})$$

$$9x_A + 6x_H \leq 1566 \quad (\text{labor})$$

$$12x_A + 16x_H \leq 2880 \quad (\text{tubing})$$

$$x_A, x_H \geq 0 \quad (\text{non-negativity})$$

Optimal value: \$66,100

Perfect Matchings

Given a bipartite graph $G = (L \cup R, E)$ define the following Linear Program:

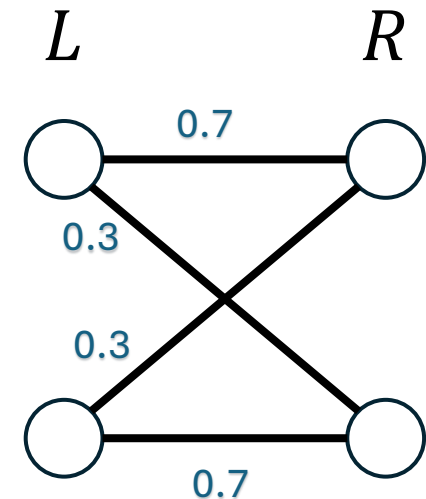
$$\min \sum_{e \in E} c_e x_e$$

such that:

$$\sum_{e \in \delta(u)} x_e = 1 \quad \text{for all } u \in L$$

$$\sum_{e \in \delta(v)} x_e = 1 \quad \text{for all } v \in R$$

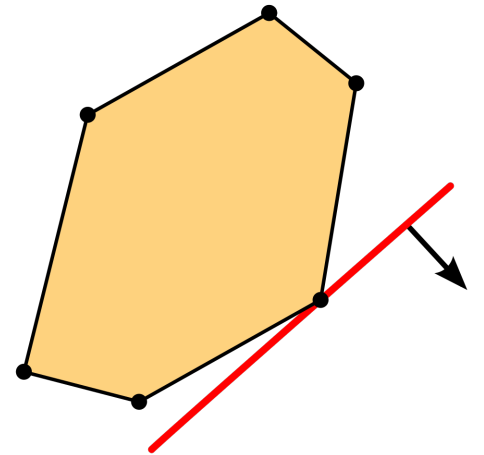
$$x_e \geq 0 \quad \text{for all } e \in E$$



The above Linear Program is a **perfect formulation**.

Linear Programming

- A very general problem in P. Already models many problems
- Used as a subroutine in modern algorithm design
 - Approximation Algorithms
 - Online Algorithms
- An expressive language for all optimization problems



Duality



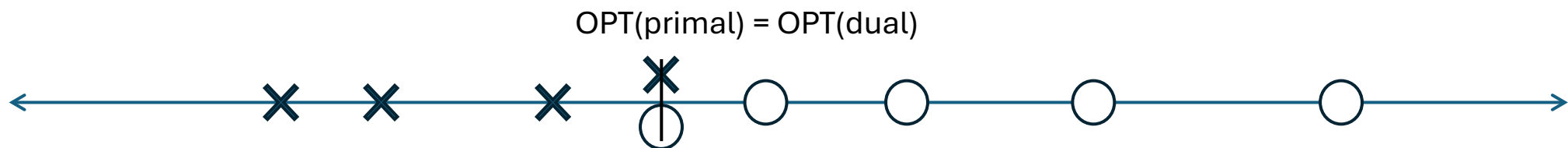
Duality

All Linear Programs come in dual pairs. One maximizes one minimizes.

Weak Duality: given a primal LP and its dual LP:

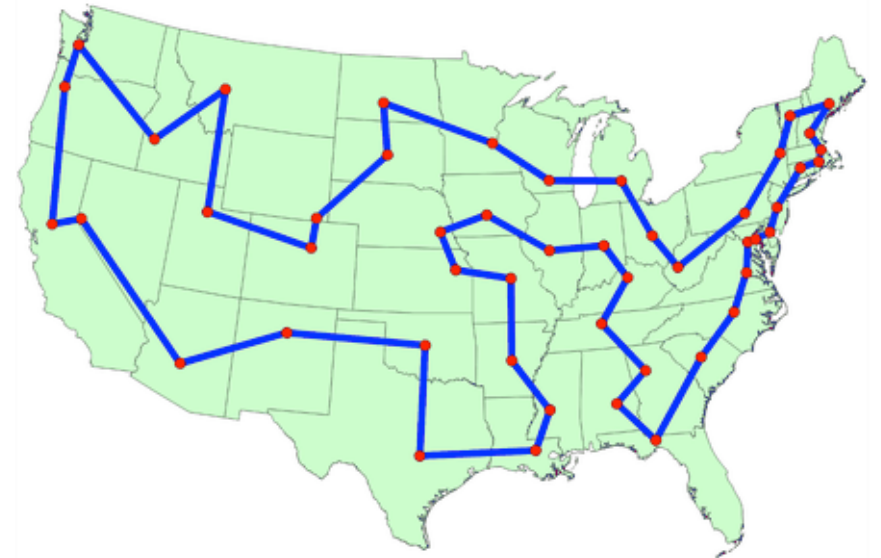
objective value of any
feasible solution to the
maximization problem \leq objective value of any
feasible solution to the
minimization problem

Strong Duality: if primal has an optimal solution, then so does its dual, and their optimal values are **equal**.



Dealing with NP-hard problems

- Restricting the class of instances under consideration (Vertex-Cover in bipartite graphs, Metric TSP)
- Exponential time algorithms
 - Parameterized Algorithms
 - Dynamic Programming
- Approximation algorithms
 - Scheduling
 - Clustering
 - Traveling Salesperson
 - Vertex Cover



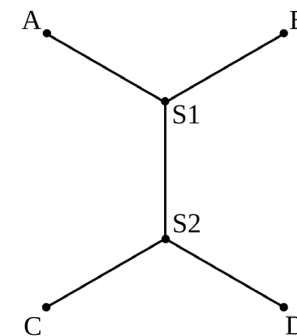
Online Algorithms

We must make decisions **online** without full information.



Consider:

- Ski-rental, Online caching
- Matching users to rides, ad slots to advertisers
- Maintain connectivity in a growing network



Maintain a solution which is competitive with the **clairvoyant optimum**.

Online Algorithms

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Techniques:

- Better-late-than never
- Caching analysis
- Primal-dual analysis
- **Randomization is provably useful**



New Directions

- You are now capable of accessing and utilizing results at the forefront of algorithms
- Segue: your final project!
- Take on something yourself and share it with us

Thursday, April 30th		
	Jade	Karger-Stein Global Min-cut
	Lenny	Top Trading Cycles / YRMHIGYT
	Catherine	Fair Division
	Caleb	The Adwords problem
	Prof. Zlatin	Congestion Games and Braess' Paradox
Tuesday, May 5th		
	Selene	k-Best Matchings
	Henok and Biruk	Vickrey and GSP Auction
	Prince	Incentives in Stable Matchings
	Tiernan	Questions in Clustering
	Fede	Online Caching